AP Calculus - AB

Differentiation Review Problems

In each of Questions 1–20 a function is given. Choose the alternative that is the derivative, $\frac{dy}{dx}$, of the function.

1. $y = x^5 \tan x$

(A) $5x^4 \tan x$ (B) $x^5 \sec^2 x$ (C) $5x^4 \sec^2 x$

(D) $5x^4 + \sec^2 x$ (E) $5x^4 \tan x + x^5 \sec^2 x$

2. $y = \frac{2-x}{3x+1}$

(A) $-\frac{7}{(3x+1)^2}$ (B) $\frac{6x-5}{(3x+1)^2}$ (C) $-\frac{9}{(3x+1)^2}$

(D) $\frac{7}{(3x+1)^2}$ (E) $\frac{7-6x}{(3x+1)^2}$

3. $y = \sqrt{3-2x}$

(A) $\frac{1}{2\sqrt{3-2x}}$ (B) $-\frac{1}{\sqrt{3-2x}}$ (C) $-\frac{(3-2x)^{3/2}}{3}$

(D) $-\frac{1}{3-2x}$ (E) $\frac{2}{3}(3-2x)^{3/2}$

4. $y = \frac{2}{(5x+1)^3}$

(A) $-\frac{30}{(5x+1)^2}$ (B) $-30(5x+1)^{-4}$ (C) $\frac{-6}{(5x+1)^4}$

(D) $-\frac{10}{3}(5x+1)^{-4/3}$ (E) $\frac{30}{(5x+1)^4}$

5. $y = 3x^{2/3} - 4x^{1/2} - 2$

(A) $2x^{1/3} - 2x^{-1/2}$ (B) $3x^{-1/3} - 2x^{-1/2}$ (C) $\frac{9}{5}x^{5/3} - 8x^{3/2}$

(D) $\frac{2}{x^{1/3}} - \frac{2}{x^{1/2}} - 2$ (E) $2x^{-1/3} - 2x^{-1/2}$

6. $y = 2\sqrt{x} - \dfrac{1}{2\sqrt{x}}$

(A) $x + \dfrac{1}{x\sqrt{x}}$ (B) $x^{-1/2} + x^{-3/2}$ (C) $\dfrac{4x-1}{4x\sqrt{x}}$

(D) $\dfrac{1}{\sqrt{x}} + \dfrac{1}{4x\sqrt{x}}$ (E) $\dfrac{4}{\sqrt{x}} + \dfrac{1}{x\sqrt{x}}$

7. $y = \sqrt{x^2 + 2x - 1}$

(A) $\dfrac{x+1}{y}$ (B) $4y(x+1)$ (C) $\dfrac{1}{2\sqrt{x^2+2x-1}}$

(D) $-\dfrac{x+1}{(x^2+2x-1)^{3/2}}$ (E) none of these

8. $y = \dfrac{x^2}{\cos x}$

(A) $\dfrac{2x}{\sin x}$ (B) $-\dfrac{2x}{\sin x}$ (C) $\dfrac{2x\cos x - x^2 \sin x}{\cos^2 x}$

(D) $\dfrac{2x\cos x + x^2 \sin x}{\cos^2 x}$ (E) $\dfrac{2x\cos x + x^2 \sin x}{\sin^2 x}$

9. $y = \ln \dfrac{e^x}{e^x - 1}$

(A) $x - \dfrac{e^x}{e^x - 1}$ (B) $\dfrac{1}{e^x - 1}$ (C) $-\dfrac{1}{e^x - 1}$

(D) 0 (E) $\dfrac{e^x - 2}{e^x - 1}$

10. $y = \tan^{-1} \dfrac{x}{2}$

(A) $\dfrac{4}{4+x^2}$ (B) $\dfrac{1}{2\sqrt{4-x^2}}$ (C) $\dfrac{2}{\sqrt{4-x^2}}$

(D) $\dfrac{1}{2+x^2}$ (E) $\dfrac{2}{x^2+4}$

11. $y = \ln(\sec x + \tan x)$

(A) $\sec x$ (B) $\dfrac{1}{\sec x}$ (C) $\tan x + \dfrac{\sec^2 x}{\tan x}$

(D) $\dfrac{1}{\sec x + \tan x}$ (E) $-\dfrac{1}{\sec x + \tan x}$

12. $y = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$

(A) 0 (B) 1 (C) $\dfrac{2}{(e^x + e^{-x})^2}$

(D) $\dfrac{4}{(e^x + e^{-x})^2}$ (E) $\dfrac{1}{e^{2x} + e^{-2x}}$

13. $y = \ln\left(\sqrt{x^2+1}\right)$

(A) $\dfrac{1}{\sqrt{x^2+1}}$ (B) $\dfrac{2x}{\sqrt{x^2+1}}$ (C) $\dfrac{1}{2\left(x^2+1\right)}$

(D) $\dfrac{x}{x^2+1}$ (E) $\dfrac{2x}{x^2+1}$

14. $y = \sin\left(\dfrac{1}{x}\right)$

(A) $\cos\left(\dfrac{1}{x}\right)$ (B) $\cos\left(-\dfrac{1}{x^2}\right)$ (C) $-\dfrac{1}{x^2}\cos\left(\dfrac{1}{x}\right)$

(D) $-\dfrac{1}{x^2}\sin\left(\dfrac{1}{x}\right)+\dfrac{1}{x}\cos\left(\dfrac{1}{x}\right)$ (E) $\cos(\ln x)$

15. $y = \dfrac{1}{2\sin 2x}$

(A) $-\csc 2x \cot 2x$ (B) $\dfrac{1}{4\cos 2x}$ (C) $-4\csc 2x \cot 2x$

(D) $\dfrac{\cos 2x}{2\sqrt{\sin 2x}}$ (E) $-\csc^2 2x$

16. $y = e^{-x}\cos 2x$

(A) $-e^{-x}(\cos 2x + 2\sin 2x)$
(B) $e^{-x}(\sin 2x - \cos 2x)$
(C) $2e^{-x}\sin 2x$
(D) $-e^{-x}(\cos 2x + \sin 2x)$
(E) $-e^{-x}\sin 2x$

17. $y = \sec^2(x)$

(A) $2 \sec x$ (B) $2 \sec x \tan x$ (C) $2 \sec^2 x \tan x$

(D) $\sec^2 x \tan^2 x$ (E) $\tan x$

18. $y = x \ln^3 x$

(A) $\dfrac{3 \ln^2 x}{x}$ (B) $3 \ln^2 x$ (C) $3x \ln^2 x + \ln^3 x$

(D) $3(\ln x + 1)$ (E) none of these

19. $y = \dfrac{1+x^2}{1-x^2}$

(A) $-\dfrac{4x}{(1-x^2)^2}$ (B) $\dfrac{4x}{(1-x^2)^2}$ (C) $\dfrac{-4x^3}{(1-x^2)^2}$

(D) $\dfrac{2x}{1-x^2}$ (E) $\dfrac{4}{1-x^2}$

20. $y = \sin^{-1} x - \sqrt{1-x^2}$

(A) $\dfrac{1}{2\sqrt{1-x^2}}$ (B) $\dfrac{2}{\sqrt{1-x^2}}$ (C) $\dfrac{1+x}{\sqrt{1-x^2}}$

(D) $\dfrac{x^2}{\sqrt{1-x^2}}$ (E) $\dfrac{1}{\sqrt{1+x}}$

In each of Questions 21–24, y is a differentiable function of x. Choose the alternative that is the derivative $\dfrac{dy}{dx}$.

21. $x^3 - y^3 = 1$

(A) x (B) $3x^2$ (C) $\sqrt[3]{3x^2}$ (D) $\dfrac{x^2}{y^2}$ (E) $\dfrac{3x^2-1}{y^2}$

22. $x + \cos(x+y) = 0$

(A) $\csc(x+y) - 1$ (B) $\csc(x+y)$ (C) $\dfrac{x}{\sin(x+y)}$

(D) $\dfrac{1}{\sqrt{1-x^2}}$ (E) $\dfrac{1-\sin x}{\sin y}$

23. $\sin x - \cos y - 2 = 0$

(A) $-\cot x$ (B) $-\cot y$ (C) $\dfrac{\cos x}{\sin y}$

(D) $-\csc y \cos x$ (E) $\dfrac{2-\cos x}{\sin y}$

24. $3x^2 - 2xy + 5y^2 = 1$

(A) $\dfrac{3x+y}{x-5y}$ (B) $\dfrac{y-3x}{5y-x}$ (C) $3x+5y$

(D) $\dfrac{3x+4y}{x}$ (E) none of these

26. If $f(x) = x^4 - 4x^3 + 4x^2 - 1$, then the set of values of x for which the derivative equals zero is

(A) $\{1, 2\}$ (B) $\{0, -1, -2\}$ (C) $\{-1, +2\}$
(D) $\{0\}$ (E) $\{0, 1, 2\}$

27. If $f(x) = 16\sqrt{x}$, then $f''(4)$ is equal to

(A) -32 (B) -16 (C) -4 (D) -2 (E) $-\dfrac{1}{2}$

28. If $f(x) = \ln x^3$, then $f''(3)$ is

(A) $-\dfrac{1}{3}$ (B) -1 (C) -3 (D) 1 (E) none of these

29. If a point moves on the curve $x^2 + y^2 = 25$, then, at $(0, 5)$, $\dfrac{d^2y}{dx^2}$ is

(A) 0 (B) $\dfrac{1}{5}$ (C) -5 (D) $-\dfrac{1}{5}$ (E) nonexistent

31. If $f(x) = 5^x$ and $5^{1.002} \simeq 5.016$, which is closest to $f'(1)$?

(A) 0.016 (B) 1.0 (C) 5.0 (D) 8.0 (E) 32.0

32. If $y = e^x(x - 1)$, then $y''(0)$ equals

(A) -2 (B) -1 (C) 0 (D) 1 (E) none of these

35. $\lim\limits_{h\to 0} \dfrac{(1+h)^6 - 1}{h}$ is

(A) 0 (B) 1 (C) 6 (D) ∞ (E) nonexistent

36. $\lim\limits_{h\to 0} \dfrac{\sqrt[3]{8+h} - 2}{h}$ is

(A) 0 (B) $\dfrac{1}{12}$ (C) 1 (D) 192 (E) ∞

37. $\lim\limits_{h\to 0} \dfrac{\ln(e+h) - 1}{h}$ is

(A) 0 (B) $\dfrac{1}{e}$ (C) 1 (D) e (E) nonexistent

39. If $f(x) = \begin{cases} \dfrac{4x^2-4}{x-1}, & x \neq 1 \\ 4, & x = 1 \end{cases}$, which of these statements are true?

I. $\lim_{x\to 1} f(x)$ exists.

II. f is continuous at $x = 1$.

III. f is differentiable at $x = 1$.

(A) none (B) I only (C) I and II only

(D) I and III only (E) I, II, and III

40. If $g(x) = \begin{cases} x^2, & x \leq 3 \\ 6x-9, & x > 3 \end{cases}$, which of these statements are true?

I. $\lim_{x\to 3} g(x)$ exists.

II. g is continuous at $x = 3$.

III. g is differentiable at $x = 3$.

(A) I only (B) II only (C) III only

(D) I and II only (E) I, II, and III

41. The function $f(x) = x^{2/3}$ on $[-8, 8]$ does not satisfy the conditions of the Mean Value Theorem because

(A) $f(0)$ is not defined (B) $f(x)$ is not continuous on $[-8, 8]$
(C) $f'(-1)$ does not exist (D) $f(x)$ is not defined for $x < 0$
(E) $f'(0)$ does not exist

42. If $f(x) = 2x^3 - 6x$, at what point on the interval $0 \leq x \leq \sqrt{3}$, if any, is the tangent to the curve parallel to the secant line on that interval?

(A) 1 (B) -1 (C) $\sqrt{2}$ (D) 0 (E) nowhere

43. If h is the inverse function of f and if $f(x) = \dfrac{1}{x}$, then $h'(3) =$

(A) -9 (B) $-\dfrac{1}{9}$ (C) $\dfrac{1}{9}$ (D) 3 (E) 9

45. If $\sin(xy) = x$, then $\dfrac{dy}{dx} =$

(A) $\sec(xy)$ (B) $\dfrac{\sec(xy)}{x}$ (C) $\dfrac{\sec(xy) - y}{x}$

(D) $\dfrac{1+\sec(xy)}{x}$ (E) $\sec(xy) - 1$

In Questions 57–64, differentiable functions f and g have the values shown in the table.

x	f	f'	g	g'
0	2	1	5	–4
1	3	2	3	–3
2	5	3	1	–2
3	10	4	0	–1

57. If $A = f + 2g$, then $A'(3) =$

(A) -2 (B) 2 (C) 7 (D) 8 (E) 10

58. If $B = f \cdot g$, then $B'(2) =$

(A) -20 (B) -7 (C) -6 (D) -1 (E) 13

59. If $D = \frac{1}{g}$, then $D'(1) =$

(A) $-\frac{1}{2}$ (B) $-\frac{1}{3}$ (C) $-\frac{1}{9}$ (D) $\frac{1}{9}$ (E) $\frac{1}{3}$

60. If $H(x) = \sqrt{f(x)}$, then $H'(3) =$

(A) $\frac{1}{4}$ (B) $\frac{1}{2\sqrt{10}}$ (C) 2 (D) $\frac{2}{\sqrt{10}}$ (E) $4\sqrt{10}$

61. If $K(x) = \left(\frac{f}{g}\right)(x)$, then $K'(0) =$

(A) $\frac{-13}{25}$ (B) $-\frac{1}{4}$ (C) $\frac{13}{25}$ (D) $\frac{13}{16}$ (E) $\frac{22}{25}$

62. If $M(x) = f(g(x))$, then $M'(1) =$

(A) -12 (B) -6 (C) 4 (D) 6 (E) 12

63. If $P(x) = f(x^3)$, then $P'(1) =$

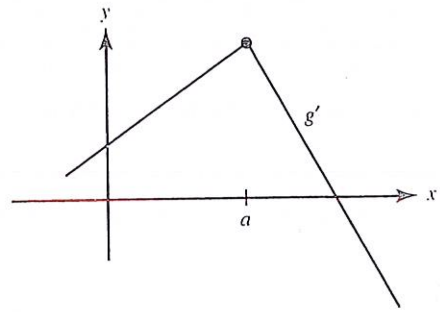
(A) 2 (B) 6 (C) 8 (D) 12 (E) 54

64. If $S(x) = f^{-1}(x)$, then $S'(3) =$

(A) -2 (B) $-\frac{1}{25}$ (C) $\frac{1}{4}$ (D) $\frac{1}{2}$ (E) 2

65. The graph of g' is shown here. Which of the following statements is (are) true of g at $x = a$?

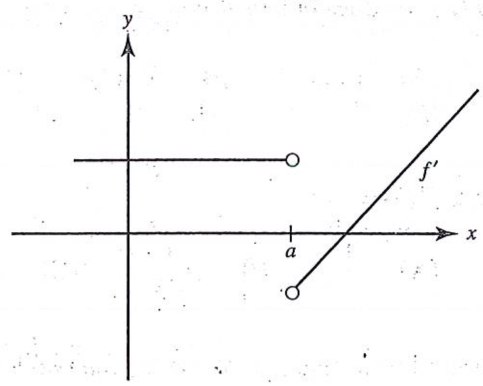
I. g is continuous.
II. g is differentiable.
III. g is increasing.



(A) I only (B) III only (C) I and III only
(D) II and III only (E) I, II, and III

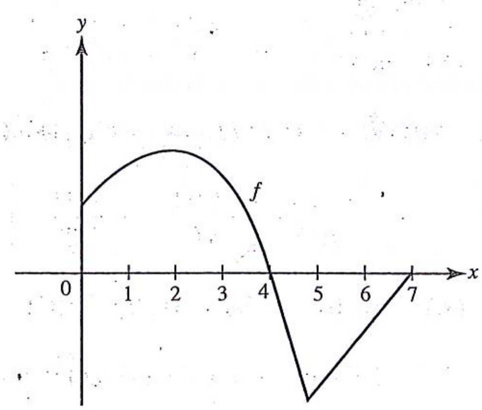
66. A function f has the derivative shown. Which of the following statements must be false?

(A) f is continuous at $x = a$.
(B) $f(a) = 0$.
(C) f has a vertical asymptote at $x = a$.
(D) f has a jump discontinuity at $x = a$.
(E) f has a removable discontinuity at $x = a$.



67. The function f whose graph is shown has $f' = 0$ at $x =$

(A) 2 only
(B) 2 and 5
(C) 4 and 7
(D) 2, 4, and 7
(E) 2, 4, 5, and 7

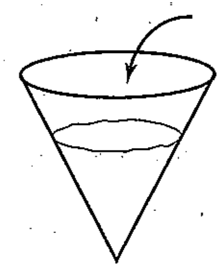


68. A differentiable function f has the values shown. Estimate $f'(1.5)$.

x	1.0	1.2	1.4	1.6
$f(x)$	8	10	14	22

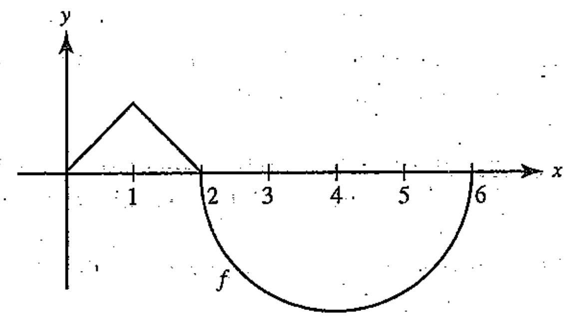
(A) 8 (B) 12 (C) 18 (D) 40 (E) 80

69. Water is poured into a conical reservoir at a constant rate. If $h(t)$ is the rate of change of the depth of the water, then h is



(A) constant
(B) linear and increasing
(C) linear and decreasing
(D) nonlinear and increasing
(E) nonlinear and decreasing

Use the figure to answer Questions 70–72. The graph of f consists of two line segments and a semicircle.



70. $f'(x) = 0$ for $x =$

(A) 1 only
(B) 2 only
(C) 4 only
(D) 1 and 4
(E) 2 and 6

71. $f'(x)$ does not exist for $x =$

(A) 1 only (B) 2 only (C) 1 and 2
(D) 2 and 6 (E) 1, 2, and 6

72. $f'(5) =$

(A) $\frac{1}{2}$ (B) $\frac{1}{\sqrt{3}}$ (C) 1 (D) 2 (E) $\sqrt{3}$

73. At how many points on the interval $[-5,5]$ is a tangent to $y = x + \cos x$ parallel to the secant line?

(A) none (B) 1 (C) 2 (D) 3 (E) more than 3

74. From the values of f shown, estimate $f'(2)$.

x	1.92	1.94	1.96	1.98	2.00
$f(x)$	6.00	5.00	4.40	4.10	4.00

(A) −0.10 (B) −0.20 (C) −5 (D) −10 (E) −25

77. The table below shows some points on a function f that is both continuous and differentiable on the closed interval [2,10].

x	2	4	6	8	10
$f(x)$	30	25	20	25	30

Which must be true?

(A) $f(x) > 0$ for $2 < x < 10$
(B) $f'(6) = 0$
(C) $f'(8) > 0$
(D) The maximum value of f on the interval [2,10] is 30.
(E) For some value of x on the interval [2,10] $f'(x) = 0$.

78. If f is differentiable and difference quotients overestimate the slope of f at $x = a$ for all $h > 0$, which must be true?

(A) $f'(a) > 0$ (B) $f'(a) < 0$ (C) $f''(a) > 0$
(D) $f''(a) < 0$ (E) none of these

83. If $f(x) = \dfrac{1}{x^2+1}$ and $g(x) = \sqrt{x}$, then the derivative of $f(g(x))$ is

(A) $\dfrac{-\sqrt{x}}{(x^2+1)^2}$ (B) $-(x+1)^{-2}$ (C) $\dfrac{-2x}{(x^2+1)^2}$

(D) $\dfrac{1}{(x+1)^2}$ (E) $\dfrac{1}{2\sqrt{x}(x+1)}$

84. If $f(a) = f(b) = 0$ and $f(x)$ is continuous on $[a, b]$, then

(A) $f(x)$ must be identically zero
(B) $f'(x)$ may be different from zero for all x on $[a, b]$
(C) there exists at least one number c, $a < c < b$, such that $f'(c) = 0$
(D) $f'(x)$ must exist for every x on (a, b)
(E) none of the preceding is true

85. Suppose $y = f(x) = 2x^3 - 3x$. If $h(x)$ is the inverse function of f, then $h'(-1) =$

(A) -1 (B) $\dfrac{1}{5}$ (C) $\dfrac{1}{3}$ (D) 1 (E) 3

86. Suppose $f(1) = 2$, $f'(1) = 3$, and $f'(2) = 4$. Then $(f^{-1})'(2)$

(A) equals $-\dfrac{1}{3}$ (B) equals $-\dfrac{1}{4}$ (C) equals $\dfrac{1}{4}$

(D) equals $\dfrac{1}{3}$ (E) cannot be determined

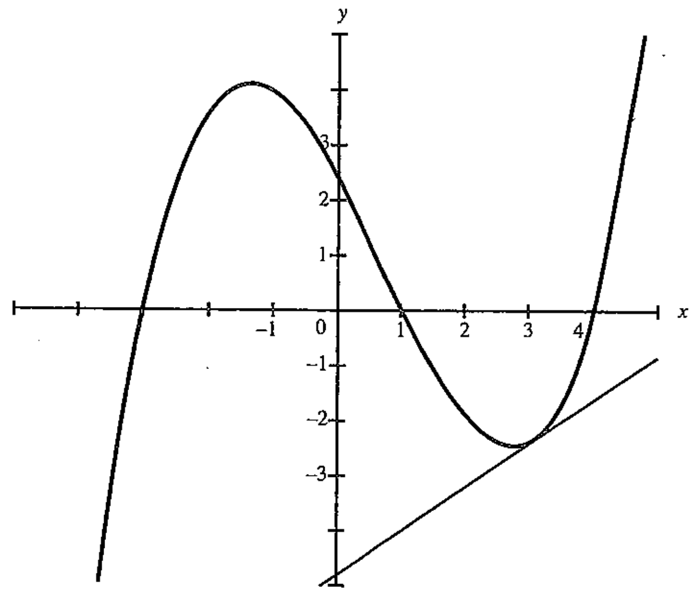
87. If $f(x) = x^3 - 3x^2 + 8x + 5$ and $g(x) = f^{-1}(x)$, then $g'(5) =$

(A) 8 (B) $\dfrac{1}{8}$ (C) 1 (D) $\dfrac{1}{53}$ (E) 5

88. Suppose $\lim_{x \to 0} \frac{g(x) - g(0)}{x} = 1$. It follows necessarily that

(A) g is not defined at $x = 0$
(B) g is not continuous at $x = 0$
(C) the limit of $g(x)$ as x approaches 0 equals 1
(D) $g'(0) = 1$
(E) $g'(1) = 0$

Use this graph of $y = f(x)$ for Questions 89 and 90.



89. $f'(3)$ is most closely approximated by

(A) 0.3 (B) 0.8 (C) 1.5 (D) 1.8 (E) 2

90. The rate of change of $f(x)$ is least at $x \approx$

(A) –3 (B) –1.3 (C) 0 (D) 0.7 (E) 2.7

Use the following definition of the *symmetric difference quotient for* $f'(x_0)$ for Questions 91–93: For small values of h,

$$f'(x_0) = \frac{f(x_0 + h) - f(x_0 - h)}{2h}.$$

91. For $f(x) = 5^x$, what is the estimate of $f'(2)$ obtained by using the symmetric difference quotient with $h = 0.03$?

(A) 25.029 (B) 40.236 (C) 40.252 (D) 41.223 (E) 80.503

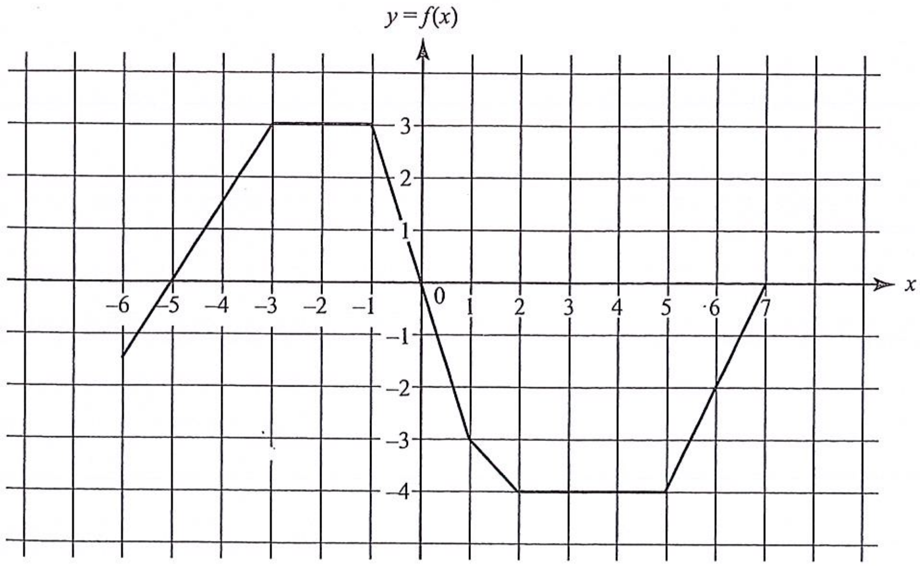
92. To how many places is the symmetric difference quotient accurate when it is used to approximate $f'(0)$ for $f(x) = 4^x$ and $h = 0.08$?

(A) 1 (B) 2 (C) 3 (D) 4 (E) more than 4

95. If $\frac{d}{dx}f(x) = g(x)$ and $h(x) = \sin x$, then $\frac{d}{dx}f(h(x))$ equals

(A) $g(\sin x)$ **(B)** $\cos x \cdot g(x)$ **(C)** $g'(x)$
(D) $\cos x \cdot g\,(\sin x)$ **(E)** $\sin x \cdot g(\sin x)$

Questions 97–101 are based on the following graph of $f(x)$, sketched on $-6 \leqslant x \leqslant 7$. Assume the horizontal and vertical grid lines are equally spaced at unit intervals.



97. On the interval $1 < x < 2, f(x)$ equals

(A) $-x - 2$ **(B)** $-x - 3$ **(C)** $-x - 4$ **(D)** $-x + 2$ **(E)** $x - 2$

98. Over which of the following intervals does $f'(x)$ equal zero?

I. $(-6,-3)$ II. $(-3,-1)$ III. $(2,5)$

(A) I only **(B)** II only **(C)** I and II only
(D) I and III only **(E)** II and III only

99. How many points of discontinuity does $f'(x)$ have on the interval $-6 < x < 7$?

(A) none **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

100. For $-6 < x < -3, f'(x)$ equals

(A) $-\frac{3}{2}$ **(B)** -1 **(C)** 1 **(D)** $\frac{3}{2}$ **(E)** 2

101. Which of the following statements about the graph of $f'(x)$ is false?

(A) It consists of six horizontal segments.
(B) It has four jump discontinuities.
(C) $f'(x)$ is discontinuous at each x in the set $\{-3,-1,1,2,5\}$.
(D) $f'(x)$ ranges from -3 to 2.
(E) On the interval $-1 < x < 1, f'(x) = -3$.